RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 44, NUMBER 17 1 NOVEMBER 1991-I

Anyonic superconductivity in a modified large-U Hubbard model

Asim Gangopadhyaya
Department of Physics, Loyola University, Chicago, Illinois 60626

Prasanta K. Panigrahi
*Laboratoire de Physique Nucléaire, Université de Montréal, Montréal, Québec, Canada H3C 3J7**
and Department of Physics, University of Illinois at Chicago, Chicago, Illinois 60680
(Received 22 July 1991)

A modified large-U Hubbard model at half filling is analyzed by a mean-field approach. Preserving a local U(1) symmetry of the action, the fluctuations about half filling are studied in the spirit of the commensurate-flux-phase condition. The fluctuations then contribute a Chern-Simons term to the tree level Lagrangian with a coefficient appropriate to that of a half fermion. With the Coulomb repulsion term, we study the low-energy excitations of the model and show the existence of superconductivity in the presence of a four-Fermi interaction term.

Strongly correlated electronic systems on a plane, in particular, the Hubbard and related models, have received considerable attention in the recent literature[1] due to their relevance to the copper oxide superconductors. In particular, after Laughlin and co-worker's observations[2] regarding the possible connection between high-T_c superconductivity and the fractional Hall effect, continuum theories with the statistics-altering Chern-Simons (CS) term have been analyzed exhaustively.[3] This approach has led to the idea of anyonic superconductivity with its parity (P) and time-reversal (T) symmetry-breaking ground state properties. Although it is far from clear that this form of superconductivity is indeed realized in nature, finding the connection between continuum CS theories and the lattice models is of interest on its own merit and might have relevance for other planar materials.

In this paper we study a U(1) subgroup of the local SU(2) symmetry of the large-U limit of the half-filled Hubbard model in the presence of additional frustration terms, described by Affleck *et al.*[4] Unlike the slave-boson technique,[5] this method does not separate the spin and charge degrees of freedom. The fluctuations are considered assuming the commensurate flux phase[6] (CFP) to be the ground state, leading to the presence of an Abelian CS term in the Lagrangian. The Coulomb repulsion term is included to prohibit large fluctuations and thereby ensure the stability of the CFP. We treat the fluctuations of the statistical gauge field as nonuniform and study the effective action involving the statistical and the physical photon field. We find that an additional repulsive four-Fermi interaction is essential in this approach for the existence of a stable massless mode, the carrier of the supercurrent. The four-Fermi term with a coefficient $d > 1/m$, m being the mass of the fermions, contributes at the level of the second derivative in the effective action. In the case of large m and in the long-wavelength limit, this term contributes only to the kinetic energy of the fluctuations and not to the coefficient of the Chern-Simons term, and, hence, is not expected to alter the ground state.

The antiferromagnetic Heisenberg model, the Mott limit of the Hubbard model at half filling, is given by

$$H = J\sum_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j \, . \qquad (1)$$

This Hamiltonian is invariant under global spin SU(2) transformations and also under a local SU(2) symmetry that leaves the spin operator $\mathbf{S}$, given by $\mathbf{S} = \mathrm{tr}\Psi^\dagger \Psi \boldsymbol{\sigma}^T$, invariant. Here $\boldsymbol{\sigma}^T$ is the transpose of $\boldsymbol{\sigma}$, and the field Ψ, given by

$$\Psi \equiv \begin{pmatrix} c_1 & c_2 \\ c_2^\dagger & -c_1^\dagger \end{pmatrix},$$

transforms under the local SU(2) as $\Psi_i \to h(x)\Psi_i$, with $h(x)$ being a 2×2 unitary matrix. At half filling the Lagrangian of the theory, $L = \frac{1}{2}\sum_i \mathrm{tr}\Psi_i^\dagger (id/dt)\Psi_i - H$, when supplemented by the one-particle-per-lattice-site (OPPLS) constraint $\frac{1}{2}\sum_i \mathrm{tr}\Psi_i^\dagger a_{0i}\Psi_i$, is invariant under space-time-dependent gauge transformations.[4] Here a_{0i} ($a_{0i} = \frac{1}{2}\mathbf{a}_{0i}\cdot\boldsymbol{\tau}$) is a Lagrange multiplier that transforms as the time component of the gauge field. The equations of motion for $\mathbf{a}_{0i}$, written succinctly as $\mathrm{tr}\Psi_i^\dagger \boldsymbol{\tau}\Psi_i = 0$, imply no vacant site, no double occupancy, and only one particle per lattice site. Clearly the first two conditions are redundant once the third one is given, which suggests a diagonal U(1) will suffice to implement the OPPLS condition.

For finite values of U, the coefficient of the on-site Coulomb term, a deviation from half filling was accomplished in Ref. 4 by adding a term $\frac{3}{8}U\sum_i a_{0i}^2$, which is not invariant under time-dependent SU(2) transformations. The chemical-potential term to move away from half filling further reduces the symmetry to that of a local time-independent U(1) subgroup of SU(2). To have a gauge-invariant theory in the continuum we choose to gauge the U(1) subgroup of SU(2), and take into account the Coulomb term in a gauge-invariant manner.

The Hamiltonian of Eq. (1) can be written in terms of the field Ψ and the Hubbard-Stratonovich matrix valued field U_{ij} of Ref. 4 as

$$H = \frac{8}{J}\sum_{i,j} \mathrm{tr}U_{ij}^\dagger U_{ij} + \sum_{i,j} \mathrm{tr}[\Psi_i^\dagger U_{ij}\Psi_j] \, ,$$

where $U_{ij}=\frac{1}{8}J\Psi_i^\dagger\Psi_j$. Explicitly, $U_{ij}=\frac{1}{8}J\begin{pmatrix}-\chi_{ij}^\dagger & 0\\ 0 & \chi_{ij}\end{pmatrix}$, where χ_{ij}'s are given by $\langle c_i^{a\dagger}c_{ja}\rangle$ at a saddle point. The $|\chi|$ is assumed to acquire a nonzero constant value at the saddle point.[7] The Hamiltonian written in terms of the electron operators is given by

$$H=\frac{8}{J}\sum_{i,j}\mathrm{tr}U_{ij}^\dagger U_{ij}+\sum_{i,j}[-c_i^{a\dagger}\chi_{ij}^\dagger c_{a,j}+c_{a,i}\chi_{ij}c_j^{a\dagger}]\,.$$

Modulo a constant term, this Hamiltonian resembles that of electrons hopping in the presence of a magnetic field (since χ_{ij} at the saddle point can be written as $\chi_{ij}=|\chi|e^{ia_{ij}}$) analyzed recently by many authors.[8,9] In the large-N mean-field approach, several numerical[9] and a semiclassical[8] analyses found the ground state of this theory to be CFP, where the ground-state value of the flux per plaquette is equal to the filling fraction for a square lattice. We assume that Coulomb repulsion term does not destabilize this ground state.

It is in this spirit that we incorporate the fluctuations about the half filling and modify the OPPLS condition to $\frac{1}{2}\,\mathrm{tr}\Psi_i^\dagger\tau_3\Psi_i+b_i=0$, or in terms of the electron operators

$$\sum_a c_i^{a\dagger}c_{i,a}-1+b_i=0\,, \tag{2}$$

where b_i is a local field. In presence of the Coulomb repulsion term, the partition function can be written as

$$Z=\int Da_0Db_i\exp i\int\left[\mathcal{L}_{\mathrm{old}}+a_{0i}b_i-\tfrac{1}{2}U\sum_i b_i^2+\tfrac{1}{2}\mu\,\mathrm{tr}(\Psi^\dagger\tau_3\Psi)\right]. \tag{3}$$

So far b_i is a local field measuring the deviation from OPPLS, and should be so interpreted only on the average. First integration over the field a_{0i} generates the modified OPPLS condition and then integrating over b_i we reproduce the on-site Coulomb term $U(c_i^{a\dagger}c_{i,a}-1)^2$ (modulo a term which modifies the chemical potential).

In the CFP phase on a square lattice the filling fraction ν and the flux per plaquette ϕ are related by $\nu=\pm\phi$, i.e., $\frac{1}{2}\sum_a c_i^{a\dagger}c_{i,a}=\pm\phi$, where ϕ is measured in units of ϕ_0 ($\phi_0=hc/e\rightarrow 2\pi$, where $e=c=\hbar=1$. Near half filling in the CFP ground state the filling factor ν, modulo one flux per plaquete, is given by $(\frac{1}{2}\pm\tilde{b}/2\pi)$, where $\tilde{b}/2\pi$ measures the deviation of flux about flux-state value and is related to the gauge fields a_i phase of bond operator χ_{ij} by $\tilde{b}=\partial_1a_2-\partial_2a_1$. Now assuming that there is a gap in the spectrum, we can adiabatically shrink this flux into infinitesimally thin vortex tubes at the electron site without changing the qualitative picture of the system (see Nori, Abrahams, and Zimanyi in the Ref. 9). Comparing this CFP condition with the modified OPPLS of Eq. (2), one finds that their consistency requires an identification of the local field b with the fluctuations in the flux about the flux-state value, $\tilde{b}/\pi$. Hence the local field b_i will be interpreted as being proportional to the magnetic flux corresponding to the spatial components of a_{0i}. The term a_0b_i in the Lagrangian becomes the gauge invariant (modulo a total divergence) (CS) term $(1/2\pi)\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho$ in the continuum limit (albeit, in the Coulomb gauge).[10] It is interesting to note that the CFP condition and the modified OPPLS condition have fixed the coefficient of this CS term, and that in turn implies semionic statistics[2,11] for excitations in the continuum theory.

Assuming the fluctuation to be small and using the notation of Ref. 7, we write[10] $\chi_{\mathbf{r},\mathbf{r}+\hat{x}}=(-1)^{|\mathbf{r}|}iJ\sigma_{\hat{x}}e^{ig\tilde{a}_{\hat{x}}}$ and $\chi_{\mathbf{r},\mathbf{r}+\hat{y}}=J\sigma_{\hat{y}}e^{ig\tilde{a}_{\hat{y}}}$, where $|\mathbf{r}|=0$ (1) if $\mathbf{r}$ belongs to an even (odd) lattice. Here $\tilde{a}_{\hat{e}}$ represents the fluctuation part of the gauge field that lies on the link $\mathbf{r}-(\mathbf{r}+\hat{\mathbf{e}})$. We can then, for the spin-up component, write the Hamiltonian as

$$H^{(1)}=-2\sigma J\sum_{x\,\in\,\mathrm{even}}c_{1\mathbf{r}}^\dagger[-i(e^{-ia_{\hat{x}}}c_{1\mathbf{r}+\hat{x}}+e^{ia_{\hat{x}}}c_{1\mathbf{r}-\hat{x}})+e^{-ia_{\hat{y}}}c_{1\mathbf{r}+\hat{y}}+e^{ia_{\hat{y}}}c_{1\mathbf{r}-\hat{y}}]+\mathrm{H.c.} \tag{4}$$

Taking the other spin component into account, we get the continuum Lagrangian as[10]

$$L(\mathbf{r})=i\int d^2r\sum_a\bar{\Psi}^{(a)\dagger}\gamma^\mu(\partial_\mu-ia_\mu\tau_3)\Psi^{(a)}\,, \tag{5}$$

where $\gamma^0=\gamma_0=\sigma_3$, $\gamma^1=-i\sigma_1$, $\gamma^2=i\sigma_2$; and

$$\psi_2^{(1)}\equiv\begin{pmatrix}c_{2,o}^{(1)\dagger}\\ c_{2,e}^{(1)\dagger}\end{pmatrix},\quad \psi_2^{(2)}\equiv\begin{pmatrix}c_{2,e}^{(2)\dagger}\\ c_{2,o}^{(2)\dagger}\end{pmatrix},$$

and

$$\Psi^{(a)}\equiv\begin{pmatrix}\psi_1^{(a)}\\ \psi_2^{(a)}\end{pmatrix}.$$

The constant $4\sigma J$ has been absorbed in the velocity of the excitations.

The Coulomb term, given by $\frac{1}{2}U\sum_{i\,\in\,\mathrm{all\ sites},a}\times(c_i^{a\dagger}c_{ia}-1)^2$ goes into $\frac{1}{2}\tilde{U}b^2$ in continuum. In the presence of a weak next-to-nearest-neighbor (NNN) antiferromagnetic interaction, Wen, Wilczek, and Zee found that a parity-odd mass term can be generated in the continuum[12] and the corresponding phase is known as the chiral spin liquid state. Furthermore, if the coefficient of the NNN term J' is about half that of the nearest neighbor term, i.e., $J'\gtrsim\frac{1}{2}J$, then the chiral spin liquid is favored as the ground state. However, they did not consider the Coulomb repulsion term in their analysis. We assume their result to be valid in the presence of the U term. Collecting all the terms, the action representing the dynamics at half filling is given by

$$S=\int d^3x\left[\sum_a\bar{\Psi}_ai\gamma^\mu(\partial_\mu-ia_\mu\tau^3)\Psi_a+m\bar{\Psi}_a\Psi_a-\frac{\epsilon_{\mu\nu\rho}a^\mu\partial^\nu a^\rho}{2\pi}-\tfrac{1}{2}\tilde{U}b^2\right]. \tag{6}$$

The parameter $\tilde{U}$ is the continuum limit of the parameter U introduced on lattice. In the presence of a nondynamical electromagnetic field, the partition function can be written as

$$Z=\int[D\Psi_a][D\bar{\Psi}_a][Db][Da_0]\exp i\int d^3x\,\mathcal{L}\,, \tag{7}$$

where

$$\mathcal{L}=\sum_a \bar{\Psi}_a i\gamma^\mu[\partial_\mu - i(a_\mu + A_\mu)\tau^3]\Psi_a + m\bar{\Psi}_a\Psi_a - \frac{\epsilon_{\mu\nu\rho}a^\mu\partial^\nu a^\rho}{2\pi} - \tfrac{1}{2}\tilde{U}b^2 . \quad (8)$$

The electromagnetic field couples to the fluctuation about half filling. Assuming the field $b(x)$ to be nonuniform, we show that the system can exhibit superconductivity, if an additional four-Fermi interaction is present. Up to one loop, the partition function is given by[13]

$$Z=\int [Da_0][Da_i]\delta(\partial_i a_i)\exp\left[iS_{\rm eff} - i\int d^3x\left(\frac{\epsilon_{\mu\nu\rho}}{2\pi}a^\mu\partial^\nu a^\rho + \tfrac{1}{2}\tilde{U}b^2\right)\right], \quad (9)$$

where up to two derivatives of fields

$$S_{\rm eff}=\int d^3x\left[\frac{-m}{|m|}\frac{\epsilon_{\mu\nu\rho}}{2\pi}(a^\mu + A^\mu)\partial^\nu(a^\rho + A^\rho) + \frac{1}{6\pi|m|}(f_{\mu\nu}+F_{\mu\nu})^2\right]. \quad (10)$$

We first analyze the first-derivative terms in $S_{\rm eff}$. Now the integration over a_0 can be done yielding

$$Z \approx \int [Db]\delta\{f(b)\}, \quad (11)$$

where

$$f(b)=\left[\left(1+\frac{m}{|m|}\right)b + \frac{m}{|m|}B\right].$$

The integration over the b field can be done, after neglecting the term $a_i\partial_0 A_i$. This is justified because substituting $a_i(x) = -\epsilon_{ij}(\partial_j/\nabla^2)b(x)$ and integrating by parts, this term $\sim(1/\nabla^2)b(x)\partial_0\partial_i\partial_0 A_i$; it can therefore be neglected for a constant external electric field. A careful analysis of $f(b)$ reveals that for small B, the free energy ($F=-\ln Z$) is minimized when $m<0$, which ensures the cancellation of the tree level CS with the one generated by the quantum correction. Notice that for the opposite sign of $\tilde{b}$ in the constraint [Eq. (2)], $m>0$ will be favored and the cancellation still occurs. Only a partial cancellation occurs at finite temperature leading to semisuperconductivity;[14] the coefficient of CS term is given by $-(1/2\pi)(m/|m|)(\tanh[\frac{1}{2}\beta(\mu-m)] - \tanh[\frac{1}{2}\beta(\mu+m)])$, where $\beta=1/kT$ and μ is the chemical potential.

Analysis of the second-derivative term reveals that the positive sign in front of the $(f_{\mu\nu})^2$ term in Eq. (10) is opposite to that of the Maxwellian action and will lead to the wrong sign for the kinetic-energy term of the massless mode.

One of the possible solutions that does not require any new degree of freedom is to introduce an additional repulsive four-Fermi interaction $-d\int d^3x(\bar{\psi}\gamma_\mu\tau_3\psi)(\bar{\psi}\gamma^\mu\tau_3\psi)$, with $d>0$ among quasiparticles. We have found that the Coulomb term and a ferromagnetic frustration introduced on the lattice will generate this term in the continuum. The partition function, with the four-Fermi term, becomes[13]

$$Z=\int [Da_\mu]\exp(iS), \quad (12)$$

where

$$S=\int d^3x[-d'(f_{\mu\nu}+F_{\mu\nu})^2 - (1/2\pi)\epsilon^{\mu\nu\rho}(a_\mu\partial_\nu A_\rho + A_\mu\partial_\nu a_\rho) - \tfrac{1}{2}\tilde{U}b^2];$$

where $d'=(d/4\pi^2 - 1/6\pi|m|)$ and should be positive for a propagating mode. Since $|m|\sim J'|\chi'|$ and $J'\gtrsim\frac{1}{2}J$, the positivity of d' implies $d>4\pi/3J|\chi'|$, which can be easily satisfied by a small d, and hence we do not expect the four-Fermi term, contributing to the fluctuations, to greatly affect the ground state obtained from the nearest- and the next-to-nearest-neighbor interactions. Scaling b and the speed of propagation c, so that $\tilde{b}=\sqrt{\kappa}b$ and $\tilde{c}=\sqrt{\kappa}c$, where $\kappa^2=1+\tilde{U}/2d'>1$; and after neglecting other unimportant higher derivative terms,

$$Z=\int Da_\mu \exp i\int[-d'f_{\mu\nu}^2 - (1/2\pi) \times e^{\mu\nu\rho}(a_\mu\partial_\nu\tilde{A}_\rho + \tilde{A}_\mu\partial_\nu a_\rho)], \quad (13)$$

where $\tilde{A}_0\sqrt{\kappa}=A_0$ and $\tilde{A}_i=A_i$. Following Banks and Lykken,[3] we can write this integral as

$$Z=\int D\phi \exp i\int d^3x\frac{1}{2d'}\left(\partial_\mu\phi - \frac{1}{4\pi}\tilde{A}_\mu\right)^2. \quad (14)$$

The equation of motion for the ϕ field is given by

$$\frac{\kappa^{-1}}{c^2}\partial^0\partial_0\phi - \nabla^2\phi - \frac{1}{4\pi}\partial^\mu\tilde{A}_\mu = 0,$$

where one clearly sees that the propagation speed for the massless field ϕ has changed. The field ϕ transforms like the phase of a condensate field $\chi=|\chi_0|e^{i8\pi\phi}$:

$$\mathcal{L}=(\partial^\mu + 2i\tilde{A}^\mu)\chi^*(\partial_\mu - 2i\tilde{A}_\mu)\chi . \quad (15)$$

The ground-state value of $|\chi_0|=[32\pi^2 d']^{-1/2}$. First we observe that the charge of the condensate came naturally to be $2e$; this is a consequence of the way we implemented the OPPLS condition. We also find the mass of the A_0 field to be equal to $4|\chi_0|^2\kappa^{-1}$, whereas the mass of the A_i field is $4|\chi_0|^2$, however, without the knowledge of the parameter d, the coefficient of the four-Fermi term, we cannot say much about the penetration depth.

To summarize, we studied anyonic superconductivity in large-U Hubbard model at half filling, after incorporating the fluctuations about a CFP ground state. Fluctuations manifest as a CS term in the action. In a theory with purely electronic degrees of freedom and without additional interactions, the kinetic energy for the density fluctuation has a wrong sign indicating that this is not a stable mode. This problem can be rectified with the addition of a repulsive four-Fermi term, originating from a Coulomb and a ferromagnetic frustration term introduced at the lattice level. A renormalization-group analysis of this model is currently under study to check the stability with quantum corrections.

We acknowledge discussions with Professor U. Sukhatme, Professor W.-Y. Keung, Professor M. Paranjape, Professor D. Chang, and Professor G. T Bodwin. We would like to thank Professor R. Mackenzie for a critical reading of the manuscript and for many useful suggestions. We would like to thank Joe Kestel for some numerical work. One of us, A.G., would like to thank the high-energy physics group of Northwestern University for warm hospitality, and P.K.P. acknowledges partial support from DOE and NSERC (Canada).

---

*Present address.

[1]P. W. Anderson, Science **235**, 1196 (1987).

[2]V. Kalmeyer and R. B. Laughlin, Phys. Rev. Lett. **59**, 2095 (1987); R. B. Laughlin, Science **242**, 525 (1988).

[3]A. L. Fetter, C. B. Hanna, and R. B. Laughlin, Phys. Rev. B **39**, 9679 (1989); Y. H. Chen, B. I. Halperin, F. Wilczek, and E. Witten, Int. J. Mod. Phys. B **3**, 1001 (1989); Y. Hosotani and S. Chakravarty, Phys. Rev. B **42**, 342 (1990); T. Banks and J. D. Lykken, Nucl. Phys. **B336**, 500 (1990); P. K. Panigrahi, R. Ray, and B. Sakita, Phys. Rev. B **42**, 4036 (1990).

[4]I. Affleck, Z. Zou, T. Hsu, and P. W. Anderson, Phys. Rev. B **38**, 745 (1988).

[5]Y. Hasegawa, O. Narikiyo, K. Kuboki, and H. Fukuyama, J. Phys. Soc. Jpn. **59**, 822 (1990); P. B. Wiegmann, Phys. Rev. Lett. **65**, 2070 (1990).

[6]P. B. Wiegmann (unpublished); P. W. Anderson, B. S. Shastry, and D. Hristopulos, Phys. Rev. B **40**, 8939 (1989).

[7]J. B. Marston and I. Affleck, Phys. Rev. B **39**, 11 538 (1989).

[8]R. Rammal and J. Bellisard (unpublished).

[9]Y. Hasegawa, P. Lederer, T. M. Rice, and P. B. Wiegmann, Phys. Rev. Lett. **63**, 907 (1989); F. Nori, E. Abrahams, and G. T. Zimanyi, Phys. Rev. B **41**, 7227 (1990); also see Anderson, Shastry, and Hristopulos in Ref. 6.

[10]For details see A. Gangopadhyaya and P. Panigrahi (unpublished).

[11]F. Wilczek, Phys. Rev. Lett. **48**, 1144 (1982); R. Mackenzie and F. Wilczek, Int. J. Mod. Phys. **3**, 2827 (1988).

[12]X. G. Wen, F. Wilczek, and A. Zee, Phys. Rev. B **39**, 11 413 (1989).

[13]K. S. Babu, A. Das, and P.K. Panigrahi, Phys. Rev. D **36**, 3725 (1987); S. Deser and A. N. Redlich, Phys. Rev. Lett. **61**, 1541 (1988).

[14]J. E. Hetrick, Y. Hosotani, and B. H. Lee (unpublished).