# $K_L$-$K_S$ Mass Difference and Supersymmetric Left-Right–Symmetric Theories

Asim Gangopadhyaya
*Department of Physics, City College of New York, New York, New York 10031*
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The supersymmetric contributions to the $K_L$-$K_S$ mass difference makes the previously obtained bounds on the right-handed scale ($M_R > 1.6$ TeV) much weaker. This raises the interesting possibility that the left-right model could be tested as an alternative to $SU_L(2) \otimes U(1)$ at low energies. Also we find that to demand that the supersymmetric contribution to the $K_L$-$K_S$ mass difference be less than $3.5 \times 10^{-15}$ GeV requires that scalar-quark masses be more than 400 GeV.

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Despite all the phenomenal success, the standard model[1] has problems with aesthetics for having built in an asymmetry towards handedness. One viable alternative is the $SU_L(2) \otimes SU_R(2) \otimes U(1)_{B-L}$ model[2] in which parity is a good symmetry of the Lagrangian, and is broken spontaneously at some relatively higher scale.

The signature of the $K_L$-$K_S$ mass difference ($\Delta M_K$) has played a crucial role in constraining such breaking scales. Beall, Bander, and Soni[3] showed that $\Delta M_K$ had the wrong sign unless $M_{W_R} \gtrsim 20 M_{W_L}$. Then Chang *et al.*[4] discovered that the calculation of Ref. 3 was not complete and gauge invariance required that many more diagrams be included. However, it was shown[5] that the numerical constraint itself is not very much affected by these graphs, although conceptually it is very important to include all of them.

Recently, minimal supersymmetric (SUSY) standard models based on supergravity have been proposed,[6] which can automatically be generalized to arrive at SUSY versions of left-right (L-R) symmetric models. However, as is well known, SUSY brings in many new fields and interactions and, hence, one in general expects that the constraints of non-SUSY models may not be valid.

In this Letter I show that this is indeed the case. The new arrivals, gluino box diagrams, contribute to $\Delta M_K$ for a wide range of values of the masses of these new fields, with a sign opposite to that of the left-right box diagram. Thus, they cut into the effectiveness of the L-R model to provide the above stringent constraint. Hence, constraints obtained[3] on $M_{W_L}/M_{W_R}$ become much weaker. This raises the interesting possibility that the distinction of the model from $SU_L(2) \otimes U(1)$ could be tested at low energies. Also, I show that the SUSY contribution to $\Delta M_K$ is too large unless the scalar-quark (s-quark) masses are greater than 400 GeV. The dependence on gluino mass is found to be rather weak for a wide range of s-quark masses.

Major SUSY contributions to $\Delta M_K$ come from new flavor-changing s-quark–gluino–quark interactions. To derive the explicit form of such interaction, from here on I work with a minimal model based on supergravity. Following the procedure developed by Duncan[7] and using the fact that renormalization-group equations are left-right symmetric we get following form of the down–s-quark mass matrix:

$$m_{\tilde{d}}^2 = \begin{bmatrix} \mu^2 1 + m_d^2 + C m_u^2 & A m_g m_d \\ A m_g m_d & \mu^2 1 + m_d^2 + C m_u^2 \end{bmatrix}, \qquad (1)$$

where Hermiticity of quark masses (dictated by L-R symmetry) has been used. $A$ is the soft[8] SUSY-breaking parameter induced by supergravity. $C$ is related to the one-loop correction to the s-quark mass. Since $\mu$ is of the order of several gigaelectronvolts and other terms are proportional to the quark masses, a near degeneracy of s-quark masses is predicted. Unlike $SU_L \otimes U(1)$ theories, here diagonal blocks are identical.[9] This reduces the number of parameters involved.

The relevant interaction term of the Lagrangian can now be written down as

$$\mathscr{L}_I(\lambda) = g_3 \tilde{d}_a^{0*} \bar{\lambda}^B T_{ab}^B d_b^0.$$

Here $d^0$ and $\tilde{d}^0$ stand for quark and s-quark weak-interaction eigenstates. $\lambda$ is the gluino field. $B$ and $a$ are generator and color indices, respectively. Let $\hat{U}, \hat{\hat{U}}$ and $\hat{D}, \hat{\hat{D}}$ be the unitary matrices that relate weak states to the mass eigenstates, i.e.,

$$d^0 = \hat{D} d, \quad u^0 = \hat{U} u, \quad \tilde{d}^0 = \hat{\hat{D}} \tilde{d}, \quad \tilde{u}^0 = \hat{\hat{U}} \tilde{u}.$$

In terms of physical fields the interaction term becomes

$$\mathscr{L}_I(\lambda) = g_3 \tilde{d}_a^* \hat{\hat{D}}^\dagger \hat{D} \bar{\lambda}^B T_{ab}^B d_b.$$

Here

$$d = \begin{pmatrix} d_L \\ d_R \end{pmatrix}$$

is a $2n_g$-dimensional vector with $n_g$ being the number of generations. $\hat{D}$ and $\hat{\hat{D}}$ are unitary matrices that diagonalize the mass matrices of quark and s-quark, respectively. We can, without loss of generality, choose the down-quark mass matrix to be diagonal, i.e., $\hat{D} = 1$. For $|c| \gtrsim 1$ it has been shown in the literature,[10] that the matrix of Eq. (1) is diagonalized basically by the same matrix that diagonalizes the up-quark mass (with our choice of the quark basis the Kobayaski-Maskawa matrix $K=U^{\dagger}$). In the case of two generations we find

$$D=\begin{bmatrix} K^{\dagger} & -K^{\dagger} \\ K^{\dagger} & K^{\dagger} \end{bmatrix}$$

with

$$K=\begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix}.$$

From this gluino interaction term can be written explicitly as

$$\mathscr{L}_I(\lambda)=\frac{g_3}{\sqrt{2}}\bar{\lambda}\left[\{(\tilde{d}_1^*,\tilde{d}_2^*)+(\tilde{d}_3^*,\tilde{d}_4^*)\}K\begin{pmatrix} d_R \\ s_R \end{pmatrix}+\{(\tilde{d}_1^*,\tilde{d}_2^*)-(\tilde{d}_3^*,\tilde{d}_4^*)\}K\begin{pmatrix} d_L \\ s_L \end{pmatrix}\right].$$

We shall define some integrals and functions for future needs as follows:

$$g_{\alpha\beta}=\int\frac{d^4q}{(2\pi)^4}\frac{q^2}{(q^2+m_\lambda^2)^2(q^2+m_\alpha^2)(q^2+m_\beta^2)},\quad h_{\alpha\beta}=\int\frac{d^4q}{(2\pi)^4}\frac{1}{(q^2+m_\lambda^2)^2(q^2+m_\alpha^2)(q^2+m_\beta^2)}.$$

Here $m_{\alpha,\beta}$, and $m_\lambda$ are s-quark and gluino masses, respectively. Assuming near degeneracy of s-quark masses we get

$$g_{\alpha\beta}\simeq g_{\alpha\alpha}+[i/(2\times 16\pi^2)](m_\alpha^2-m_\beta^2)\tilde{g}_\alpha,\quad h_{\alpha\beta}\simeq h_{\alpha\alpha}+[i/(2\times 16\pi^2)](m_\alpha^2-m_\beta^2)\tilde{h}_\alpha,$$

where $\tilde{g}_\alpha$ and $\tilde{h}_\alpha$ are given by

$$\tilde{g}_\alpha=\frac{(5m_\lambda^4-4m_\alpha^2m_\lambda^2-m_\alpha^4)+2(m_\alpha^2m_\lambda^2+2m_\alpha^4)\ln(m_\alpha^2/m_\lambda^2)}{(m_\alpha^2-m_\lambda^2)^4},$$

$$\tilde{h}_\alpha=\frac{(m_\lambda^4+4m_\alpha^2m_\lambda^2-5m_\alpha^4)+2(m_\alpha^2m_\lambda^2+2m_\lambda^4)\ln(m_\alpha^2/m_\lambda^2)}{m_\alpha^2(m_\alpha^2-m_\lambda^2)^4}.$$

We define two functions $F_1$ and $F_2$ of $g_{\alpha\beta}$ and $h_{\alpha\beta}$ by

$$F_1(g)=\textstyle\sum_{\alpha,\beta}g_{\alpha\beta}(-1)^{\alpha+\beta}\quad\text{where } \alpha,\beta=1,\ldots,4,\quad F_2(g)=(g_{11}-g_{12}-g_{13}+g_{14})+\text{all cyclic replacements.}$$

One finds that the functions $F_1$ and $F_2$ vanish if all s-quark masses are equal and this property implies super Glashow-Iliopoulos-Maiani cancellation.

Now let us turn to the calculation of $\Delta M_K$. The diagrams that contribute towards $H_{\text{eff}}^{\Delta S=2}(\lambda)$ are shown in Figs. 1 and 2. Fig. 1 arises from Majorana-type mass terms of the gluino and Fig. 2 is due to Dirac-type terms. From these one finds

$$\begin{aligned}H_{\text{eff}}^{\Delta S=2}=(\alpha_S^2/8\pi^2)\sin^2\theta_C\cos^2\theta_C\{&\tfrac{38}{9}F_1(g)(V_{LL}+V_{RR})-\tfrac{1}{3}m_\lambda^2F_2(h)(T_{LL}+T_{RR})-\tfrac{74}{9}m_\lambda^2F_2(h)(S_{LL}+S_{RR})\\&+S_{LR}[\tfrac{136}{9}F_1(g)+\tfrac{80}{9}F_2(g)-\tfrac{112}{9}m_\lambda^2F_1(h)]\\&+V_{LR}[-8m_\lambda^2F_1(h)+\tfrac{16}{3}m_\lambda^2F_2(h)+\tfrac{28}{3}F_1(h)]\},\end{aligned}$$

where

$$S_{AB}=(\bar{d}P_As)(\bar{d}P_Bs),\quad V_{AB}=(\bar{d}\gamma_\mu P_As)(\bar{d}\gamma_\mu P_Bs),\quad T_{AB}=(\bar{d}\sigma_{\mu\nu}P_As)(\bar{d}\sigma_{\mu\nu}P_Bs)$$

with $P_A$ and $P_B$ being the chirality projection operators.

To calculate the matrix element of this $H_{\text{eff}}^{\Delta S=2}(\lambda)$ between $K^0$ and $\bar{K}^0$ states we determine the matrix elements

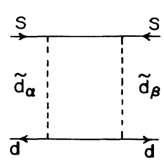


FIG. 1. Contribution to $\Delta M_K$ through Majorana mass of the gluino.

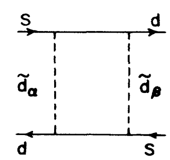


FIG. 2. Contribution to $\Delta M_K$ by Dirac-type gluino mass.

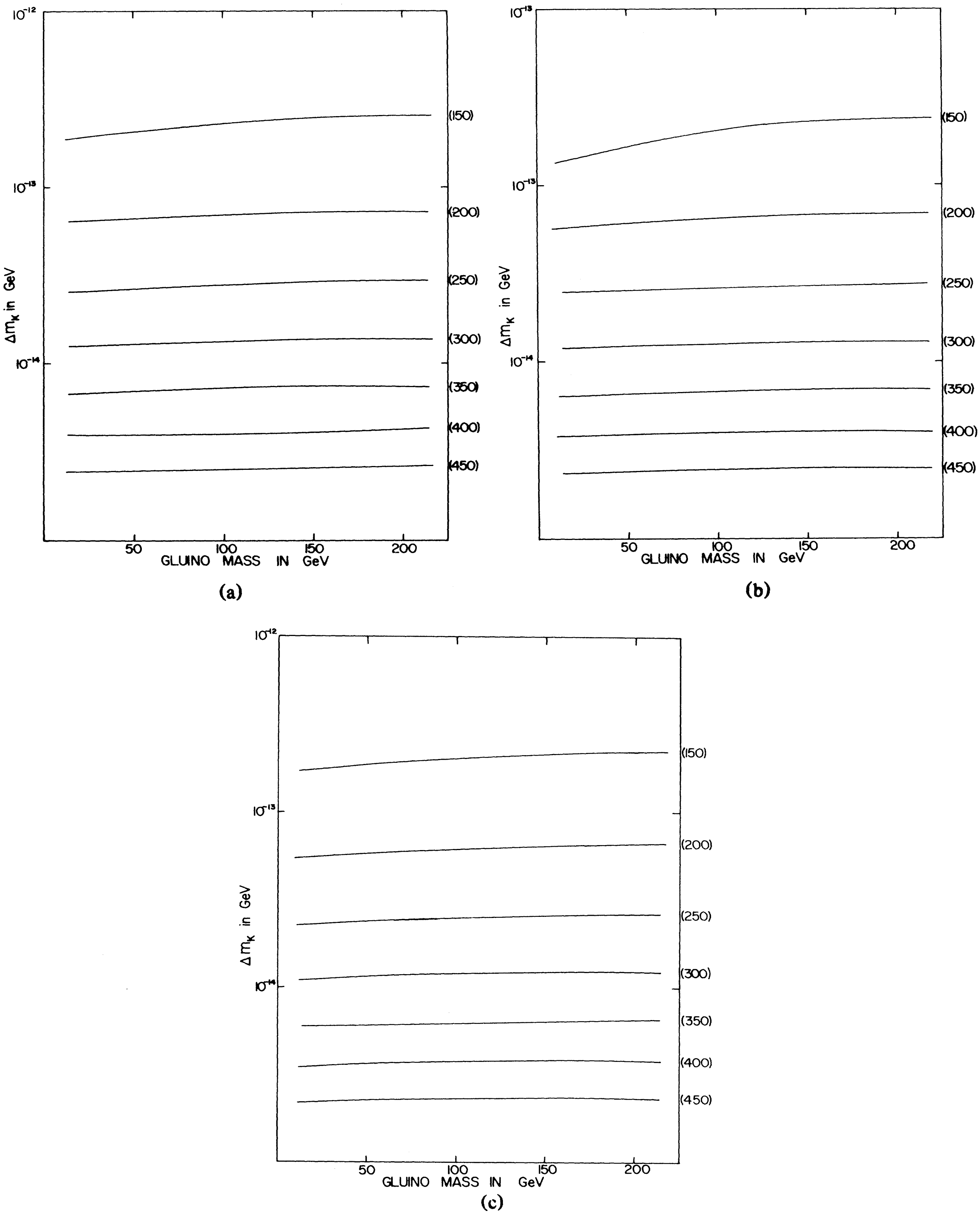


FIG. 3. $\Delta M_K$ as a function of s-quark mass, gluino mass, and gravitino mass ($m_g$). The numbers in the parentheses are s-quark masses in gigaelectronvolts. (a) $m_g = 50$ GeV; (b) $m_g = 100$ GeV, and (c) $m_g = 150$ GeV.

of the above operators by the vacuum-insertion[11] method to get

$$S_{AA}=\tfrac{5}{24}RQ,\quad S_{\rm LR}=(\tfrac{1}{24}+R/4)Q,\quad V_{AA}=Q/3,$$
$$V_{\rm LR}=-(\tfrac{1}{4}+R/6)Q,\quad T_{AA}=-RQ/4.$$

All other matrix elements vanish. Here $Q=f_KM_K$ and $R=(6\rho+1)/(4\rho-6)$, with $f_K$ and $\rho$ defined by

$$\langle 0|\bar{S}\gamma_\mu\gamma_5 d|K^0\rangle = if_Kp_\mu/(2m_K)^{1/2}$$

and

$$\langle K^0|S_{LR}|\bar{K}^0\rangle=\rho\langle K^0|V_{LR}|\bar{K}^0\rangle$$

In the vacuum-insertion approximation, one finds[3]

$$\rho=\tfrac{3}{4}M_K^2/(m_s+m_d)^2+\tfrac{1}{8}\simeq 7.7.$$

Now we shall evaluate $\Delta M_K$. Assuming near degeneracy of s-quark masses one finds

$$F_1\begin{Bmatrix}g\\h\end{Bmatrix}=4C(m_c^2-m_u^2)\begin{Bmatrix}\tilde{g}\\h\end{Bmatrix}$$

and

$$F_2\begin{Bmatrix}g\\h\end{Bmatrix}=4Am_g(m_s-m_d)\cos 2\theta_{\rm C}\begin{Bmatrix}\tilde{g}\\h\end{Bmatrix}.$$

Assuming $|C|$, $A\simeq o(1)$, as is the case in models with Polanyi-type hidden sectors, we calculate $\Delta M_K$ for a wide range of values for the masses of s-quark, gluino, and gravitino fields. The numerical result is depicted in Fig. 3, where we used $\alpha_s=0.1$, $m_u=5$ MeV, $m_c=1.5$ GeV, $m_d=25$ MeV, $m_s=150$ MeV, $f_K=0.16$ GeV, $M_K=0.5$ GeV, and $\sin\theta_{\rm C}=0.23$. The important points seen from the graphs are the following: (a) The SUSY contribution to $\Delta M_K$ has a sign opposite that of the left-right box diagram. This, as explained in the text, renders the constraint on $M_{W_L}/M_{W_R}$ much weaker. (b) The prediction for $\Delta m_K$ is much larger than the known value of $3.5\times10^{-15}$ GeV unless s-quark masses are greater than 400 GeV.

In summary, the SUSY sector of the left-right model contributes to $\Delta M_K$ with a sign such that the famous constraint on $M_R$ obtained from non-SUSY calculations[3] is rendered much weaker. This resurrects the hope that left-right models could be a nontrivial alternative to $SU_L(2)\otimes U(1)$ theory at low energies. Also, we find the magnitude of the contribution too large unless the s-quark masses are > 400 GeV.

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