POSSIBILITIES FOR FINITE GRAND UNIFICATION WITH $N = 2$ SUPERSYMMETRY

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Received 11 June 1984

We study the prospects for constructing finite grand unified theories using softly broken $N = 2$ global supersymmetry. The requirements that the supersymmetry breaking scale be of order 10^3–10^4 GeV and that the mixing between the observed light fermions and the mirror fermions present in $N = 2$ theories be small ($\ll m_W$), make it difficult to construct finite grand unified theories based on a large class of simple groups with a realistic symmetry breaking pattern.

Theories with extended supersymmetries ($N \geqslant 2$) have raised the interesting possibility for constructing finite field theories of particle interactions, and have been investigated from this point of view in several recent papers [1–3]. The highly constrained nature of such theories raises the hope of connecting the various sectors of unified gauge theories (such as the gauge fields, the Higgs bosons and matter multiplets), which would then provide a deep understanding of things that appear ad hoc in theories without supersymmetry. In order to realize this hope, one must, however, construct realistic extended supersymmetry models that are finite. In this letter, we investigate this question for $N = 2$ globally supersymmetric theories and report on our study of the prospects for constructing grand unified models based on simple or semisimple gauge group, G. We demand the model to obey the following constraints:

(I) Finiteness or $\beta(g) = 0$. This implies no U(1) factors in G.

(II) A realistic pattern of local symmetry breakdown, i.e.

$$G \to H \to \ldots SU(3) \times SU(2) \times U(1) \xrightarrow[m_W/g]{} SU(3)_c \times U(1)_{em} .$$

(III) The mirror fermions (i.e. fermions with V+A coupling to W-bosons) which are necessarily implied by $N = 2$ supersymmetry must be heavy ($m \gtrsim m_W$) and must have extremely small mixing with known light fermions.

(IV) Supersymmetry breaking is due to the inclusion of explicit soft terms (a list of which is provided in refs. [3,4]), which have a scale of no more than a TeV.

We start with a brief review of the $N = 2$ supermultiplets and the lagrangian in terms of $N = 1$ superfields.

We will be concerned with the $N = 2$ vector multiplet, which will contain the Yang–Mills fields and the hypermultiplet, which will contain matter multiplets and Higgs multiplets. In terms of $N = 1$ components, we write the vector multiplet as (V, Φ), which will be

[1] Work supported by Department of Energy.
[2] Work supported by National Science Foundation.

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(North-Holland Physics Publishing Division)

chosen to belong to the adjoint representation of the gauge group G and the hypermultiplet as (X, Y) where X and Y belong to the {R} and $\{\bar{R}\}$ representation of G. The $\hat{\Phi}, X, Y$ are chiral $N = 1$ superfields, whereas V is a $N = 1$ real vector field.

The lagrangian invariant under $N = 2$ supersymmetry can be written in terms of the above $N = 1$ superfields as follows:

$$\mathcal{L}^{(0)} = (1/8g^2)(W^\alpha W_\alpha)_F + [\sqrt{2} ig\, Y\phi X]_F + \text{h.c.} + [2\mathrm{Tr}\, \phi^\dagger e^{2gV}\phi e^{-2gV} + X^\dagger e^{2gV} X + Y^\dagger e^{-2gV} Y]_D . \quad (1)$$

W is the supersymmetric field strength $W^\alpha = \frac{1}{4}\bar{D}^2 D^\alpha V$. It is clear that the lagrangian is highly restrictive; in particular, it has no mass parameters. To study gauge symmetry breaking at the tree level, we wish to introduce soft $N = 2$ supersymmetry breaking but $N = 1$ supersymmetry preserving terms denoted by $\mathcal{L}^{(1)}$:

$$\mathcal{L}^{(1)} = m_\phi(\mathrm{Tr}\,\Phi^2)_F + \sum_{a,b} m_{ab}(X_a Y_b) , \quad (2)$$

where m_ϕ and m_{ab} are parameters which can have orders of magnitude anywhere from the weak scale to the grand unification scale. It has been already shown [4,5] that $\mathcal{L}^{(1)}$ preserves the finiteness of $N = 2$ theory.

Finally, we will add soft terms that break both supersymmetries but they will have a scale of at most a TeV or so. We would like to study the symmetry breaking in this theory so that the gauge group G breaks with or without intermediate stages down to $SU(3)_C \times SU(2)_L \times U(1)$ and then to $SU(3)_C \times U(1)_{em}$. For this purpose, we first give the representation content of the theory consistent with the finiteness condition i.e. $\beta(g) = 0$ and see whether a realistic path of descent to $SU(3)_C \times U(1)_{em}$ is realizable without giving large mass to the observed fermions.

Let us consider the grand unification symmetries to be SU(N), SO(2N), E_6, E_7 and E_8 and list the various possible finite theories. To see this, we give the formula for $\beta(g)$ in $N = 2$ theories:

$$\beta(g) = -\frac{g^3}{8\pi^2}\left(C_2(G) - \sum_R T(R)\right) , \quad (3)$$

where C_2 is the quadratic Casimir for the adjoint representation and $T(R)$, that for the representation R, with the effect of the mirror representation $\{\bar{R}\}$ included. Finiteness, therefore, implies only those set of hypermultiplets for where $C_2(G) = \Sigma_R T(R)$. We list below the allowed [‡1] choices of {R} for the various grand unification symmetries.

(1) *SU(N)*. For $N > 8$ and $N = 5$, finiteness allows only the fundamental {N}, symmetric S $[D_S = \frac{1}{2}N(N+1)]$ and antisymmetric A $[D_A = \frac{1}{2}N(N-1)]$ representations in the following combinations denoted by the number of respective multiplets present (n_F, n_S, n_A):

(a) $(2N, 0, 0)$,
(b) $(N-2, 1, 0)$,
(c) $(N+2, 0, 1)$,
(d) $(0, 1, 1)$,
(e) $(4, 0, 2)$,
(f) $(0, 0, 0)$ + 1 adjoint hypermultiplet. (4)

For $N = 6, 7$ and 8, we have the following possibilities:

(g) *SU(6)*. In this case, there is a mixed-symmetric {20}-dimensional representation, for which $T(R) = 3$. Denoting the possible sets in this case by (n_F, n_S, n_A, n_M) we have the following new possibilities in addition to the ones already listed:

$(6,0,0,1)$, $(0,0,0,2)$, $(2,0,1,1)$.

(h) *SU(7)*. The mixed symmetric representation in this case is {35}-dimensional and the new combination that arises is

$(4,0,0,1)$.

(i) *SU(8)*. The new representation in this case is {56}-dimensional, which is the totally antisymmetric third rank tensor. The new combination that leads to a finite model is (denoting by η_3 the number of {56}-dimensional representations and set by (n_F, n_S, n_A, n_3)), $(1,0,0,1)$.

(2) *SO(2N)*. Here only allowed representations (i.e. those with $T(R) \leqslant C_2$) are the fundamental (denoted by F), spinor (S) and adjoint (Ad) and the allowed combinations are: (n_F, n_S, n_{Ad})

‡1 After this work was completed, we came across a paper by Koh and Rajpoot [6], where the finite combination of representations has been given for various groups.

$$
\begin{aligned}
&\text{(a)}\ (2N-2,0,0), && N \geqslant 4,\\
&\text{(b)}\ (0,0,1), && N \geqslant 4,\\
&\text{(c)}\ (4,1,0), && N = 7,\\
&\text{(d)}\ (2,2,0),\ (6,1,0), && N = 6,\\
&\text{(e)}\ (6,1,0),\ (4,2,0),\\
&\quad\ (2,3,0),\ (0,4,0), && N = 5\,. \qquad (5)
\end{aligned}
$$

We do not discuss $N < 5$ since they are not suitable candidates for grand unification [‡2].

(3) E_6. The only allowed representations in this case are the fundamental {27} and the adjoint {78} in the following combinations (n_F, n_A)

$$
\begin{aligned}
&\text{(a)}\ (4,0)\,,\\
&\text{(b)}\ (0,1)\,. \qquad (6)
\end{aligned}
$$

(4) E_7. In the same notation as for E_6, except the fundamental is {56}-dimensional and adjoint is {33}-dimensional

$$
\begin{aligned}
&\text{(a)}\ (1,0)\,,\\
&\text{(b)}\ (0,3)\,. \qquad (7)
\end{aligned}
$$

(5) E_8. The only possible choice for the hypermultiplet is the adjoint {248}-dimensional.

We now wish to study whether the realistic chain of symmetry breaking given earlier is realizable without conflicting with other constraints (I)–(IV). Let us first analyze this in case (1), i.e. SU(N) groups.

Case (1). First we will argue that regardless of whether the fermions are assigned to the adjoint gauge Higgs representation Φ or to the hypermultiplets X, Y etc., vacuum expectation value of ϕ i.e. $\langle\phi\rangle$ must vanish or at most be $\leqslant m_W$ if it is diagonal. Next, we will argue that, with $\langle\phi\rangle \leqslant m_W$, the chain of symmetry breakdown by the hypermultiplets will not be realistic in the sense that $SU(3)_C \times SU(2)_L \times U(1)_Y$ will not emerge as a symmetry in any intermediate stage. If we choose $\langle\phi\rangle$ off-diagonal, then, large mixing between mirror and normal fermions can be avoided. Only in this case, adjoint gauge Higgs Φ could participate in symmetry breaking for $\mu \gg m_W$. However, if $\langle\phi\rangle$ is off-diagonal, it will break the group down without leaving a U(1) and will not lead to a realistic symmetry breaking chain. This will form the basis of our contention that finite, realistic grand unified SU(N) models appear difficult to construct.

To prove the first point (i.e. $\langle\phi\rangle \leqslant m_W$), consider the two cases:

(a) The known light fermions, F, are assigned to hypermultiplets (X, Y).

(b) F are assigned to the adjoint gauge Higgs.

In case (a), $\langle\Phi\rangle \neq 0$ induces fermion mass terms that mix the normal fermions with the mirror fermions. In realistic models, for the fermion mass spectrum to be acceptable, such mixing should be very small ($\leqslant m_W$) [condition (III)]. However, in certain cases, even with $\langle\Phi\rangle \neq 0$, it may be possible to avoid mixing between the mirror fermions and the light fermions F in the following way:

(i) With the appropriate fermion assignments in hypermultiplets (X, Y), and by choosing $\langle\Phi\rangle$ off diagonal, mixing can be relegated exclusively to the mirror sector of the theory.

(ii) By adding soft terms of the form mXY and fine tuning m, some of the mixing can be cancelled.

Now let us consider the possibility (i). For $\langle\Phi\rangle$ to be off diagonal, and supersymmetry to be unbroken we need two hypermultiplets, to conspire (see later).

This requirement of two hypermultiplets, combined with the finiteness condition leaves us with a choice that the symmetry be broken either by antisymmetric, A (case 1f) or by vector hypermultiplets (cases 1a, 1b, 1c, 1e). But symmetry breaking neither by the A nor the vector hypermultiplet can give rise to a U(1) factor in the residual symmetry group. Thus $\langle\Phi\rangle$ off diagonal leads to conflict with condition (II).

To consider the possibility of cancellation in the mixing between light and mirror fermions by fine tuning the coefficient of the soft term, let us note that for SU(3) × SU(2) × U(1) to be a good symmetry at some state $\langle\Phi\rangle$ must be of the form

$$\langle\Phi\rangle_{\rm dig} = (a_1, a_1, a_1, a_2, a_2, \{a_n\})\,.$$

If $a_1 \neq a_2$, then one can avoid mixing between quarks and mirror quarks or leptons and mirror leptons but not both. If $a_1 = a_2$, then $\langle\Phi\rangle$ has a stability group of at least H = SU(5) (where H may or may not contain a U(1) factor) which must be broken to SU(3) × SU(2) × U(1) using hypermultiplets. If H does not

‡2 For a review of group theory of grand unification, see ref. [7].

contain any U(1) factor, then the symmetry breaking with antisymmetric hypermultiplet A or fundamental hypermultiplet can not give rise to U(1) [‡3].

However, if H contains a U(1) factor, then there is a possibility of symmetry breaking: $H \to SU(3) \times SU(2)_L \times U(1) \times H'$. Such breaking can only be achieved with the use of an antisymmetric hypermultiplet A, (case 1f). This special case can also be ruled out by noting that after using two antisymmetric hypermultiplets to break the symmetry we are left with only four hypermultiplets in fundamental representation out of which at least one is needed to break $SU(2)_L$. Hence for the low energy symmetry to be only $SU(3) \times SU(2)_L \times U(1)$, H′ can at most be SU(3). This leaves us with the possible grand unified SU(N) group as SU(6), SU(7) or SU(8). And it is easy to convince oneself that with the above mentioned scenario in place, there are no light fermions left over in the theory which can be identified with quarks and leptons.

Case (b) (fermions in adjoint gauge Higgs Φ). In this case if $\langle\Phi\rangle \neq 0$ and if the stability group of $\langle\Phi\rangle$ is H, then the fermions in the coset space G/H are "eaten up" to form the Dirac mass for the gaugino. Only massless fermions transform as the adjoint of H. For these fermions to be identified as quarks and leptons, they must be kept massless at tree level. Hence any further symmetry breaking from H to SU(3) × SU(2) × U(1) must be done using hypermultiplets. But when the hypermultiplet (X, Y) is used to break the symmetry, the fermions in X combine with gaugino to form a massive Dirac gaugino, and fermions in Y combine with the fermions in Φ to form a massive Dirac gauge higgsino. Thus massless light fermions are the only fermions which transform as (1, 8) + (3, 1) and SU(2) × SU(3) which clearly can not be identified as quarks and leptons.

We can now show that if at a mass scale $\langle\Phi\rangle = 0$, the *most likely* tree level solution consistent with supersymmetry is $\langle X\rangle = \langle Y\rangle = 0$. To see this, we write down the F-terms following from eqs. (1) and (2). We find:

$$\partial W/\partial \phi_j^i = \sqrt{2} i g \sum_a X_i^{(a)} Y^{(a)j} + M\Phi_i^{\;i} ,$$

$$\partial W/\partial X^{i(a)} = \sum_b m_{ab} Y^{(b)i} + \sqrt{2} i g \sum_a (\Phi Y^{(a)})^i , \qquad (8)$$

$$\partial W/\partial Y^i = \sum_a m_{ab} X_i^{(a)} + \sqrt{2} i g \sum_a (X^{(a)}\Phi)_i .$$

If $m_{ab} = 0$, there is no mass scale in the theory and there are many directions in the space of the Higgs field which have $V = 0$ and either $\langle X\rangle$ or $\langle Y\rangle$ non-vanishing. It is then clear that to get the $\partial W/\partial\phi = 0$ with $\langle\phi\rangle = 0$ and the vanishing D-terms requires at least two hypermultiplets. In these cases, radiative corrections may generate a mass scale but at least at the tree level, the theory is undefined. If $m_{ab} \neq 0$, the only case when gauge symmetry can break consistent with supersymmetry is, when Det $\|M_{ab}\| = 0$. This case is also similar to the case $m_{ab} = 0$, except, there are as many flat directions as there are zero eigenvalues of the matrix m_{ab}. Again the theory is not defined at the tree level.

There are two points of view one can adopt at this stage:

(1) Radiative corrections cannot fix the vacuum expectation value at the appropriate scale. If this happens, all SU(N) models are unsuitable for grand unification since supersymmetry will force all hypermultiplets to have zero vacuum expectation value down to the scale m_W and grand unification symmetry does not break in an acceptable manner.

(2) A more optimistic point of view is to assume that even though at tree level $\langle X\rangle$ and/or $\langle Y\rangle$ remain undefined, radiative corrections/or non-perturbative effects do fix it at an acceptable value. If this happens, can one build realistic grand unified models? The important point to remember is that, in this case one needs at least two hypermultiplets, to keep supersymmetry from breaking down spontaneously. As we saw before, this makes the symmetry breaking chain unrealistic.

It is also clear from eq. (8) that if $\langle\Phi\rangle \neq 0$, for D-terms to vanish, we need at least two hypermultiplets to acquire non-vanishing VEV's, as mentioned earlier.

We therefore conclude that, within the constraints (I)–(IV), it is difficult to construct realistic finite SU(N) GUT models. It is also easy to convince one-

‡3 Symmetry breaking due to symmetric hypermultiplet S is ruled out because unbroken supersymmetry requires two symmetric hypermultiplets, which is not allowed by finiteness condition (I).

self that in case (1f), where only adjoint hypermultiplet is present, the symmetry can never break down to SU(3) × U(1). The same set of arguments also apply to cases of SU(N), N = 6, 7, 8.

Case (2). Let us now consider the SO($2N$) case. As in the SU(N) case, if the light fermions, F are part of the adjoint representation, then this multiplet must have zero vacuum expectation value. If the light fermions are part of the spinor (in analogy with the SO(10) case), then the adjoint gauge Higgs cannot acquire a VEV, otherwise the light and mirror fermions will mix, exactly as in the SU(N) case. Therefore, in cases (2b)–(2e), where the above assignment of light fermions must be done, the adjoint gauge Higgs must not participate in symmetry breaking down from SO($2N$) to $SU(3)_c \times SU(2)_L \times U(1)_Y$. We, therefore, have to study whether the remaining representations are adequate to achieve this breaking.

Let us, first, consider cases (c), (d) and (e). In these cases, the only realistic choice of multiplet for fermions is the spinor representation. The only case, which gives three or more generations is the SO(14) group, i.e. case (2c), since the spinor is {64}-dimensional for {X} and $\{\overline{64}\}$ dimensional for {Y}. Under SO(10) decomposition, they lead to 4{16} + $4\{\overline{16}\}$, as would be required to fit the observed fermions. This rules out cases (2d) and (2e). However, in case (2c), the symmetry breaking can at most occur up to SO(10) since, each fundamental representation breaks SO(N) → SO(N − 1) and no other multiplet is available to break the symmetry down further.

Coming to case (2a) where one has only fundamental representations, the only case that offers the hope of grand unification is SO(32) as noted in ref. [6], since the fundamental representation contains 16 + $\overline{16}$ under SU(16) [8]. But the pattern of symmetry breaking cannot be realistic since each fundamental representation breaks SO($2N$) → SO($2N$ − 1); therefore, there is no way that the gauge group of the standard model can emerge at the intermediate stage.

Thus, SO($2N$) unification with realistic symmetry breaking appears difficult. The arguments of this section are also applicable to SO($2N$ + 1) grand unification groups.

Case (3)–(5). The most interesting exceptional grand unification is the one based on the group E_8, where the only hypermultiplet allowed by finiteness is the adjoint representation. In this case, the theory actually becomes invariant under N = 4 supersymmetry transformation. Decomposition of the {248}-dimensional adjoint representation under SU(3) × SO(10) × U(1) group is given by

$$\begin{aligned}\{248\} \equiv & \{3,16\} + \{\bar{3},\overline{16}\} + \{3,10\} + \{\bar{3},10\} \\ & + \{3,1\} + \{\bar{3},1\} + \{8,1\} + \{1,45\} \\ & + \{1,16\} + \{1,\overline{16}\} + \{1,1\}\,,\end{aligned}$$

which is, of course, right for accommodating quarks and leptons [9]. However, there are not enough multiplets to give a realistic pattern of symmetry breakdown. Also, using the adjoint gauge Higgs to break symmetry will lead to large mixings between particles and their mirrors.

Next, let us consider E_6. Here, case (3a) where only an adjoint hypermultiplet is admitted, is unrealistic since it cannot give adequate symmetry breaking. Furthermore, it cannot accommodate more than two generations. The most promising case here is case (3b), which admits 4{27}-dimensional fundamental representations. Each {27}-dimensional representation under SO(10) contains {16} + {10} + {1}. In principle, it could yield four families of quarks and leptons. But as we will see, it is difficult to obtain the correct pattern of symmetry breakdown. The point is that, multiplets which contain light fermions F cannot participate in symmetry breaking, since otherwise the quarks and leptons get "eaten up" by gauginos. This means that at most one {27} and the four mirror $\{\overline{27}\}$ dimensional hypermultiplets can break symmetry. Since the adjoint gauge Higgs Φ must have a VEV $\langle\phi\rangle \ll m_W$ (due to arguments similar to the SU(N) and SO($2N$) case), it cannot participate in symmetry breaking. Then, we can get:

$$E_6 \xrightarrow[\{\overline{27}\}+\{27\}]{} SO(10)\,,$$

To break the symmetry down further to SU(3) × SU(2) × U(1) using $3\{\overline{27}\}$ is not possible. Thus, within our strict guidelines, this potentially interesting case also becomes unacceptable. Similar arguments also apply to the E_7 case.

In conclusion, we have studied the prospects for constructing finite grand unified theories based on unitary, orthogonal and exceptional groups with realistic pattern of symmetry breakdown. We find that, it is difficult to obtain an intermediate local symmetry corresponding to $SU(3)_c \times SU(2)_L \times U(1)_Y$ group as would be required to describe the low energy electroweak physics. Our arguments can also be easily extended to semisimple groups such as $[SU(N)]^M$ etc. without much problems. The requirement of complete finiteness would, therefore appear to be incompatible with grand unification based on a large class of interesting groups. One could, however, demand finiteness beyond the one-loop case. This may allow for more freedom in our choice of interaction terms and make the task of grand unified model building easier.

S. Kalara and A. Gangopadhyaya would like to thank the particle theory group at the University of Maryland for their hospitality.

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